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The supersymmetric (2 + 1)D noncommutative $CP^{(N-1)}$ model in the fundamental representation

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Abstract

In this paper, we study the noncommutative supersymmetric $CP^{(N-1)}$ model in 2 + 1 dimensions, where the basic field is in the fundamental representation which, differently to the adjoint representation already studied in the literature, goes to the usual supersymmetric $CP^{(N-1)}$ model in the commutative limit. We analyse the phase structure of the model and calculate the leading and subleading corrections in a $1/N$ expansion. We prove that the theory is free of non-integrable UV/IR infrared singularities and is renormalizable in the leading order. The two-point vertex function of the basic field is also calculated and renormalized in an explicitly supersymmetric way up to the subleading order.

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1. Introduction

The $CP^{(N-1)}$ model in 2 + 1 dimensions was studied since the end of the 1970 decade, mainly because it is a reasonably simple scalar model which possesses gauge invariance, and it was found to reproduce several effects typical of more complicated gauge models in four spacetime dimensions, such as instantons solutions and confinement [1, 2]. The crucial simplifying aspect of the $CP^{(N-1)}$ model is that the gauge field is non-dynamical at the classical level, its dynamics being entirely generated by quantum corrections. The possibility of studying in a simpler setting some of the most crucial aspects of gauge theories has been one of the main sources of interest in the model.

The phase structure of the $CP^{(N-1)}$ model, for example, was studied in [3], unveiling the existence of two phases. In one of them the symmetry $SU(N)$ is broken down to $SU(N - 1)$, whereas in the other the model remains $SU(N)$ symmetric and mass generation occurs for the fundamental bosonic fields. Afterwards, it was found [4] that the coupling to fermions preserves the two phases structure but the long-range force is hindered by the fermionic fields.

More recently, the supersymmetric $CP^{(N-1)}$ model was studied both in the Wess–Zumino gauge [5] as well as using a manifestly supersymmetric covariant formulation [6].

On the other hand, in the past few years there have been a great deal of interest in quantum field theories defined over a noncommutative spacetime [7]. Sources of this interest are, among others, their relations with string theory [8] and quantum gravity [9]. In particular, gauge theories defined in a noncommutative spacetime have been intensively studied, and several interesting effects were found, such as UV/IR mixing [10, 11] and strong restrictions of gauge groups and couplings to the matter [12–14]. The gauge invariance of the quantum corrections to the effective action also becomes very non-trivial in the noncommutative setting [15, 16].

Noncommutative gauge theories present a rich spectrum of phenomena, yet they can be also very complicated to deal with. So it was natural to investigate a simpler model with gauge invariance, such as the $CP^{(N-1)}$ model. When extending this model to the noncommutative spacetime, one finds more than one possibility of coupling the Lagrange multiplier and the gauge fields to the basic bosonic fields. In the non-supersymmetric case, the so-called fundamental and adjoint representations were considered in [17], using a $1/N$ expansion. In the fundamental representation, the model turned out to be renormalizable and free of dangerous infrared UV/IR singularities. However, in the adjoint representation, the appearance of non-integrable UV/IR divergences presented itself as an obstacle to the consistency of the model in higher orders of the $1/N$ expansion.

It is now well known that supersymmetry helps in avoiding the UV/IR problem in noncommutative field theories [18–20]. Indeed, a supersymmetric extension of the noncommutative $CP^{(N-1)}$ model in the adjoint representation was studied in [21], showing that the aforementioned problem is not present. However, in that case, when the noncommutativity of the space is withdrawn the model does not return to the commutative supersymmetric $CP^{(N-1)}$, but to a free scalar theory, where the basic scalar field does not satisfy a constraint nor is coupled to an auxiliary gauge field.

In the present work, we are going to analyse the supersymmetric noncommutative $CP^{(N-1)}$ model in the fundamental representation, i.e., the case where the commutative limit really goes to the usual commutative supersymmetric $CP^{(N-1)}$ model. We shall study the phase structure of the model and the issue of the UV/IR mixing in the Feynman integrals, aiming at establishing its renormalizability at the leading order in the $1/N$ expansion. We will also look at the subleading corrections to the two-point vertex function of the scalar superfield, and show how the use of an explicitly supersymmetric quantization scheme avoids some difficulties with the usual (component) approach.

Other aspects of the noncommutative supersymmetric $CP^{(N-1)}$ model in the fundamental representation were also focused in the recent literature. The structure of BPS and non-BPS solitons was first found to be quite similar to the commutative model [22–25], but then novel solutions with no commutative counterparts were found [26]. This raised questions about the equivalence between the CP^1 and the nonlinear sigma model in the noncommutative case [27]. Alternatives to the above investigations, employing the Seiberg–Witten map, can be found in [28, 29]. More recently, the dynamics of the $CP^{(N-1)}$ model in a non-anticommutative spacetime has also been investigated [30, 31].

This paper is organized as follows. In section 2, we present the model in superfield formulation and study its phase structure. Leading-order corrections to the effective action of the auxiliary and gauge superfields are calculated in section 3, and the renormalizability of the model at the leading order is discussed in section 4. In section 5, the quadratic effective action of the scalar superfield is discussed at the subleading order. Finally, section 6 contains our

conclusions and final remarks. In the appendix, we explicitly give the component formulation of the superfield model studied by us.

2. The noncommutative supersymmetric $CP^{(N-1)}$ model

The bosonic $CP^{(N-1)}$ model, in commutative spacetime, when the matter field is in the fundamental representation, is defined by the action [3]

$$S = \int d^3x \left\{ \overline{D^\mu \phi_a} D_\mu \phi_a + \sigma \left(\phi_a \bar{\phi}_a - \frac{N}{g_0} \right) \right\}, \tag{1}$$

where ϕ_a is an N -uple of scalar fields and σ is a scalar Lagrange multiplier which enforces the constraint $\phi \bar{\phi} = N/g_0$. The covariant derivative is $D^\mu = \partial^\mu - iA^\mu$, A_μ being an auxiliary vector gauge field, which classically is the composite field $A^\mu = i\phi \overleftrightarrow{\partial}^\mu \bar{\phi}/2\phi\bar{\phi}$. The spacetime index μ runs over 0, 1, 2, and we use the metric $g^{\mu\nu} = (-, +, +)$.

The model defined by (1) can be generalized to a noncommutative spacetime where coordinates satisfy

$$[x^\mu, x^\nu] = i\Theta^{\mu\nu} \tag{2}$$

by substituting into (1) the usual product of functions by the *-Moyal product [32]. The divergence structure of the noncommutative $CP^{(N-1)}$ model has been extensively discussed in [17, 21]. Here we shall focus on a noncommutative supersymmetric extension of the model (1) which, adopting the conventions of [33], is given by

$$S = - \int d^5z \left\{ \frac{1}{2} \overline{\nabla^\alpha \Phi_a} * \nabla_\alpha \Phi_a + \Sigma * \left(\Phi_a * \bar{\Phi}_a - \frac{N}{g_0} \right) \right\}. \tag{3}$$

In this expression, $z = (x^\mu, \theta^\alpha)$, where x^μ with $\mu = 0, 1, 2$ and θ^α with $\alpha = 1, 2$ are, respectively, the bosonic and the grassmanian superspace coordinates, Φ_a is an N -uple of scalar superfields, Σ is a scalar Lagrange multiplier superfield, and A_α is a two-component spinor auxiliary gauge superfield. The supercovariant spinorial derivative is given by $\nabla_\alpha = D_\alpha - iA_\alpha$, where $D_\alpha = \partial_\alpha + i\theta^\beta \partial_{\beta\alpha}$. We remark that the *-Moyal product in (3) is defined by [36]

$$f(x, \theta) * g(x, \theta) = \exp \left(\frac{i}{2} \Theta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y^\nu} \right) f(x, \theta) g(y, \theta)|_{x=y}, \tag{4}$$

affecting only the bosonic coordinates x^μ , which are noncommutative, whereas the grassmanian coordinates satisfy the usual anticommutativity rule $\{\theta^a, \theta^b\} = 0$.¹ To avoid troubles with unitarity, we shall restrict the noncommutativity to the spatial bosonic coordinates, what amounts to consider $\Theta^{0i} = 0$ [37]².

The action above is $U(N)$ globally invariant and $U(1)$ gauge invariant. The infinitesimal gauge transformations are given by

$$\begin{aligned} \Phi_a &\rightarrow \Phi'_a = (1 + iK) * \Phi_a, \\ \bar{\Phi}_a &\rightarrow \bar{\Phi}'_a = \bar{\Phi}_a * (1 - iK), \\ A_\alpha &\rightarrow A'_\alpha = A_\alpha + D_\alpha K + i[K, A_\alpha]_*, \\ \Sigma &\rightarrow \Sigma' = \Sigma + i[K, \Sigma]_*, \end{aligned} \tag{5}$$

¹ Note, however, that one can consider the extension of the noncommutativity to the grassmanian superspace coordinates in four spacetime dimensions [34]. Recently, this idea has also been extended to three-dimensional spacetime [35].

² It is interesting to remember that unitarity violations are a peculiar aspect of the approach for noncommutative field theories we consider. There are alternative proposals which do not suffer from this issue, see [9, 38–40].

where K is a real scalar superfield. The ordering of Φ and $\bar{\Phi}$ in the constraint term in equation (3) implies in the need of Σ being transformed under the gauge transformation in order to maintain the invariance of the action (observe that the transformation of Σ disappears when we take the commutative limit $\Theta \rightarrow 0$, as it should). Had we chosen the opposite order for Φ and $\bar{\Phi}$, then Σ would not need to transform, but a mixing between the fields Σ and A^α would appear already in leading $1/N$ approximation (we will return to this point later).

By writing the original (unrenormalized) superfields in terms of renormalized ones through the definitions, $\Phi = Z_1^{1/2}\Phi_R$, $A^\alpha = Z_3 A_R^\alpha$ and $\Sigma = Z_2 \Sigma_R$, the action gets written as

$$S = - \int d^5 z \left\{ \frac{Z_1}{2} D^\alpha \bar{\Phi}_{Ra} D_\alpha \Phi_{Ra} + Z_2 \Sigma \left(Z_1 \Phi_{Ra} \bar{\Phi}_{Ra} - \frac{N}{g_0} \right) - \frac{i Z_1 Z_3}{2} (D_\alpha \bar{\Phi}_{Ra} A_{R\alpha} \Phi_{Ra} + \bar{\Phi}_{Ra} A_{Ra} D^\alpha \Phi_{Ra}) + \frac{1}{2} Z_1 Z_3^2 \bar{\Phi}_{Ra} A_R^2 \Phi_{Ra} \right\}. \quad (6)$$

In this equation and in the remaining of this paper, we will not explicitly indicate the *-Moyal product, which should be understood to be present when multiplying fields in configuration space. Wavefunctions and coupling constant counterterms are defined through

$$\begin{aligned} Z_1 &= 1 + \delta_\Phi, \\ Z_1 Z_3 &= (1 + \delta_\Phi)(1 + \delta_A) = 1 + \delta_e, \\ Z_1 Z_3^2 &= (1 + \delta_\Phi)(1 + \delta_A)^2 = 1 + \delta_b, \\ Z_1 Z_2 &= (1 + \delta_\Phi)(1 + \delta_\Sigma) = 1 + \delta_c, \\ Z_2/g_0 &= \mu/g + \delta_g, \end{aligned} \quad (7)$$

where μ is an arbitrary parameter with the dimension of mass and g is the adimensional renormalized coupling constant. Substituting these expressions in equation (6) and omitting the subindex R that indicates renormalized fields, we have

$$S = - \int d^5 z \left\{ \frac{1}{2} D^\alpha \bar{\Phi}_a D_\alpha \Phi_a + \Sigma \left(\Phi_a \bar{\Phi}_a - \frac{N\mu}{g} \right) + \frac{\delta_\Phi}{2} D^\alpha \bar{\Phi}_a D_\alpha \Phi_a - i \frac{\delta_e}{2} (D^\alpha \bar{\Phi}_a A_\alpha \Phi_a + \bar{\Phi}_a A_\alpha D^\alpha \Phi_a) + \frac{\delta_b}{2} \bar{\Phi}_a A^\alpha A_\alpha \Phi_a - N \delta_g \Sigma + \delta_c \Sigma \Phi_a \bar{\Phi}_a \right\} \quad (8)$$

From now on, we will not explicitly write the R subindex; all fields will be understood to be the renormalized ones.

To study the phase structure of the model let us suppose that the superfields Σ and Φ acquire constant non-vanishing vacuum expectation values (VEVs) $\langle \Sigma(x, \theta) \rangle = m$ and $\langle \Phi(x, \theta) \rangle = \sqrt{N}v$, this last one, for simplicity, supposed to be in the $a = N$ component³. These VEVs will play the role of order parameters identifying the different phases we will find. By redefining the fields in term of new fields that have zero VEVs,

$$\begin{aligned} \Phi_a(x, \theta) &\longrightarrow \bar{\Phi}_a(x, \theta), & a = 1, \dots, N-1, \\ \Phi_N(x, \theta) &\longrightarrow \Phi_N(x, \theta) + v\sqrt{N}, \\ \bar{\Phi}_N(x, \theta) &\longrightarrow \bar{\Phi}_N(x, \theta) + \bar{v}\sqrt{N}, \\ \Sigma(x, \theta) &\longrightarrow \Sigma(x, \theta) + m, \\ A_\alpha(x, \theta) &\longrightarrow A_\alpha(x, \theta), \end{aligned} \quad (9)$$

³ We shall not consider possible quantum phases that could arise from classical noncommutative solitonic solutions, which can be singular at small Θ [41].

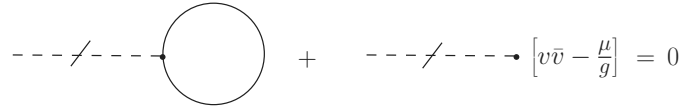


Figure 1. Gap equation for the Σ superfield, including the tadpole loop correction. In this and in the following figures, solid lines represent Φ_a propagators. Dashed cut lines represent the Σ field.

the action of equation (8) is written as

$$\begin{aligned}
 S = \int d^5z \left\{ \bar{\Phi}_a (D^2 - m) \Phi_a - \Sigma \left(\Phi_a \bar{\Phi}_a - \frac{N\mu}{g} + Nv\bar{v} \right) - \frac{1}{2} \bar{\Phi}_a A^\alpha A_\alpha \Phi_a \right. \\
 + \frac{i}{2} [D^\alpha \bar{\Phi}_a A_\alpha \Phi_a + \bar{\Phi}_a A_\alpha D^\alpha \Phi_a + v\sqrt{N} D^\alpha \bar{\Phi}_N A_\alpha - \bar{v}\sqrt{N} A^\alpha D_\alpha \Phi_N] \\
 - \frac{\sqrt{N}}{2} \bar{v} A^\alpha A_\alpha \Phi_N + \frac{\sqrt{N}}{2} v \bar{\Phi}_N A^\alpha A_\alpha + \frac{N}{2} \bar{v} v A^\alpha A_\alpha - m\sqrt{N} (v\bar{\Phi}_N + \bar{v}\Phi_N) \\
 \left. - \Sigma \Phi_N \bar{v}\sqrt{N} + \Sigma \bar{\Phi}_N v\sqrt{N} \right\} + S_{CT}, \tag{10}
 \end{aligned}$$

where S_{CT} is a short for the counterterms action, and $D^2 \equiv \frac{1}{2} D^\alpha D_\alpha$. The propagator for the first $(N - 1)$ components of Φ_a is given by

$$\langle T \Phi_a(p, \theta_1) \bar{\Phi}_b(-p, \theta_2) \rangle = -i \delta_{ab} \left(\frac{D^2 + m}{p^2 + m^2} \right) \delta_{12}, \tag{11}$$

where $\delta_{12} \equiv \delta^2(\theta_1 - \theta_2)$. The interaction vertices of the theory are

$$\begin{aligned}
 \Sigma \Phi \bar{\Phi} &\longrightarrow -i e^{-ik_2 \wedge k_3} \Sigma(1) \Phi(2) \bar{\Phi}(3), \\
 \bar{\Phi} A^\alpha D_\alpha \Phi + \dots &\longrightarrow \frac{1}{2} e^{-ik_2 \wedge k_3} A^\alpha(1) D_\alpha [\Phi(2) \bar{\Phi}(3)], \\
 \bar{\Phi} A^\alpha A_\alpha \Phi &\longrightarrow \frac{i}{2} \cos(k_1 \wedge k_2) e^{-ik_3 \wedge k_4} A^\alpha(1) A_\alpha(2) \Phi(3) \bar{\Phi}(4),
 \end{aligned} \tag{12}$$

where $k \wedge p \equiv \frac{1}{2} k_\mu \Theta^{\mu\nu} p_\nu$ and $A^\alpha(1)$ denotes $A^\alpha(k_1, \theta)$, and similarly for the other fields.

The condition that, in leading order of $1/N$, the redefined fields Σ and Φ have zero vacuum expectation values implies in the equations,

$$i \int_\epsilon \frac{d^3k}{(2\pi)^3} \frac{1}{k^2 + m^2} + \frac{\mu}{g} + \delta_g(\epsilon) - v\bar{v} = 0, \quad m\bar{v} = mv = 0, \tag{13}$$

where ϵ in the integral symbol and in δ_g represent an ultraviolet regulator. The first gap equation is represented graphically in figure 1. These equations are the same as the corresponding ones for the supersymmetric commutative model [5]. The dependence on Θ of the underlying noncommutativity of the spacetime, manifested through the phase factors appearing in the vertices, disappears due to the vanishing of the momentum entering through the external leg of Σ or Φ_N . In particular, this fact ensures that UV/IR infrared singularities do not appear in the gap equation. The scalar integral in (13) can be performed using dimensional reduction [42], leading to

$$i \int_\epsilon \frac{d^3k}{(2\pi)^3} \frac{1}{k^2 + m^2} = \frac{|m|}{4\pi}, \tag{14}$$

hence, the counterterm δ_g turns out to be finite, providing an arbitrary finite renormalization of the gap equation, which now reads

$$\frac{|m|}{4\pi} + \frac{\mu}{g} + \delta_g - v\bar{v} = 0, \quad m\bar{v} = mv = 0. \quad (15)$$

One convenient choice for the counterterm is $\delta_g = -\mu/4\pi$, where $\mu > 0$ is the same mass scale introduced in (7), so that the solution of the first of equations (15) can be written as

$$g = \frac{\mu}{\bar{v}v + \frac{\mu - |m|}{4\pi}}. \quad (16)$$

From the second of equations (15) we see that the model presents two phases:

- (i) a broken $U(N)$ phase in which Φ_N has a vacuum expectation value $\langle \Phi_N \rangle = N^{1/2}v \neq 0$ and the fields Φ remain massless ($m = 0$). This happens for

$$g = \frac{4\pi}{(1 + 4\pi\bar{v}v/\mu)} < 4\pi. \quad (17)$$

- (ii) A symmetric phase in which Φ has an induced mass $m \neq 0$ but $\langle \Phi \rangle = 0$. This happens for

$$g = \frac{4\pi}{1 - |m/\mu|} > 4\pi. \quad (18)$$

From equation (18) one can immediately calculate the β function in the symmetric phase,

$$\beta(g) \equiv \mu \frac{dg}{d\mu} = 4\pi \frac{|m/\mu|}{(1 - |m/\mu|)^2} = g \left(1 - \frac{g}{4\pi} \right). \quad (19)$$

As can be read from this formula, β goes to zero for $\mu \rightarrow \infty$, characterizing an ultraviolet fixed point at $g = 4\pi$. This result is the same as that for the corresponding commutative model [5]. The same analysis for the behaviour of the β function in the broken phase leads to similar conclusions.

We stress that the choice $\delta_g = -\mu/4\pi$ is convenient, but not essential. Any value of δ_g provided that $\delta_g < 0$ leads to the same phase structure, only the value of the critical g changes. If $\delta_g > 0$, on the other hand, the symmetric phase does not exist, while for $\delta_g = 0$, the model does not have the broken phase. Finally, one could choose other regularization to calculate the divergent integral in equation (14), in which case it could be necessary the counterterm δ_g to contain an infinite renormalization to render the gap equation (13) finite. Even in this case, the above considerations would apply without any changes.

3. Effective propagators at leading order

As we are mainly interested in studying the renormalization of the model and the two phases have the same ultraviolet behaviour [43] we will work in the symmetric phase from now on, so that the action reads

$$S = \int d^5z \left\{ \bar{\Phi}_a (D^2 - m) \Phi_a - \Sigma \left(\Phi_a \bar{\Phi}_a - \frac{N\mu}{g} \right) - \frac{1}{2} \bar{\Phi}_a A^\alpha A_\alpha \Phi_a + \frac{i}{2} [D^\alpha \bar{\Phi}_a A_\alpha \Phi_a + \bar{\Phi}_a A_\alpha D^\alpha \Phi_a] \right\} + S_{CT}, \quad (20)$$

where g is related to m and μ by equation (18), and

$$S_{CT} = \int d^5z \left\{ \delta_\Phi \bar{\Phi}_a D^2 \Phi_a - m \delta_c \bar{\Phi}_a \Phi_a - i \frac{\delta_e}{2} (D^\alpha \bar{\Phi}_a A_\alpha \Phi_a + \bar{\Phi}_a A_\alpha D^\alpha \Phi_a) - \frac{\delta_b}{2} \bar{\Phi}_a A^\alpha A_\alpha \Phi_a + N \delta_g \Sigma - \delta_c \Sigma \Phi_a \bar{\Phi}_a \right\}. \quad (21)$$

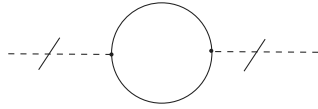


Figure 2. Leading order of $1/N$ contribution to the effective propagator of the Σ field.

From these equations, we see that the propagator of Φ_N is also given by equation (11). This fact simplifies the perturbative calculations in the next subsections.

3.1. *The two point effective action of the Σ field*

As we read from equation (20), classically the Σ field is purely a constraint field without dynamics. However, in leading order of $1/N$ it acquires a (nonlocal) kinetic term becoming a propagating field. In this approximation, the radiative corrections to its two point effective action (see figure 2) is given by

$$\Gamma_{\Sigma}^{(2)} = \frac{1}{2} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta \Sigma(-p, \theta) [N f(p)(D^2 + 2m)] \Sigma(p, \theta). \tag{22}$$

The exponential factors in the two vertices cancel between themselves and the result is similar to the commutative one. The nonlocal character of this action is explicit in the factor $f(p)$,

$$\begin{aligned} f(p) &= \int \frac{d^3 k}{(2\pi)^3} \frac{1}{[(k+p)^2 + m^2](k^2 + m^2)} \\ &= -\frac{1}{4\pi\sqrt{p^2}} \operatorname{arctg} \left(\frac{1}{2} \sqrt{\frac{p^2}{m^2}} \right) = \begin{cases} -1/8|p| & \text{for } p \rightarrow \infty \\ -1/8\pi m & \text{for } p \rightarrow 0. \end{cases} \end{aligned} \tag{23}$$

The finiteness of $\Gamma_{\Sigma}^{(2)}$ is consistent with the absence of a Σ^2 counterterm in the original action. From equation (22), we arrive at the following Σ propagator,

$$\langle T \Sigma(p, \theta_1) \Sigma(-p, \theta_2) \rangle = \frac{i}{N} \frac{(D^2 - 2m)}{f(p)(p^2 + 4m^2)} \delta_{12}, \tag{24}$$

which is regular in the infrared ($p^2 \rightarrow 0$) while decreasing as $1/p$ in the ultraviolet limit ($p^2 \rightarrow \infty$).

Since, by definition, the Σ effective propagator is *minus* the inverse of the kernel in equation (22), in the supergraph formalism we still have the identity represented graphically in figure 3, which is known to hold in the usual commutative $CP^{(N-1)}$ model [3]. This powerful identity is very important in the study of subleading quantum corrections to the vertex functions of the model, as we will comment in section 5.

3.2. *The two point effective action of the spinorial gauge potential*

Another field whose dynamics is generated only at the quantum level is the spinorial superfield A_{α} . The leading $1/N$ radiative correction to its two point effective action is represented in figure 4. The contribution of the figure 4(a) gives

$$\Gamma_{4a}^{(2)} = \frac{N}{2} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta \int_{\epsilon} \frac{d^3 k}{(2\pi)^3} \frac{C^{\alpha\beta}}{k^2 + m^2} A_{\alpha}(p, \theta) A_{\beta}(-p, \theta), \tag{25}$$

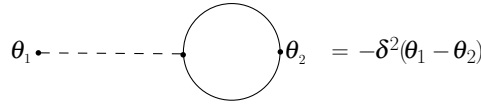


Figure 3. This supergraph identity, discussed in [3] in the non-supersymmetric $CP^{(N-1)}$ model, secures the cancellation of several divergences when the perturbation theory is expanded to include one particle reducible graphs. Without this identity, the proof of renormalizability of the $CP^{(N-1)}$ model at higher orders becomes unfeasible.

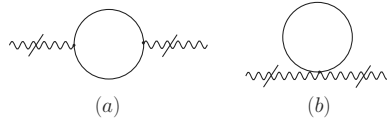


Figure 4. Leading order of $1/N$ contribution to the spinorial superfield effective propagator. Wavy cut lines represent the A field.

while figure 4(b) yields

$$\Gamma_{4b}^{(2)} = -\frac{N}{2} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta \int_{\epsilon} \frac{d^3 k}{(2\pi)^3} \frac{1}{[(k+p)^2 + m^2](k^2 + m^2)} \times \left[(k^2 + m^2)C^{\alpha\beta} + (k^{\alpha\beta} + mC^{\alpha\beta})D^2 + \frac{1}{2}(k^{\gamma\beta} + mC^{\gamma\beta})D_{\gamma}D^{\alpha} \right] \times A_{\alpha}(p, \theta)A_{\beta}(-p, \theta). \tag{26}$$

Adding the two contributions above we get

$$\Gamma_4^{(2)} = -\frac{N}{2} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta \int_{\epsilon} \frac{d^3 k}{(2\pi)^3} \frac{1}{[(k+p)^2 + m^2](k^2 + m^2)} (k^{\alpha\beta} + mC^{\alpha\beta}) \times \left(D^2 A_{\alpha}(p, \theta)A_{\beta}(-p, \theta) + \frac{1}{2}D_{\alpha}D^{\gamma}A_{\gamma}(p, \theta)A_{\beta}(-p, \theta) \right). \tag{27}$$

Individually, the graphs in figure 4 are linearly ultraviolet divergent, but their leading divergences cancel between themselves, so that $\Gamma_4^{(2)}$ contains at most logarithmic divergences. In fact, it turns out to be finite, since $\int_{\epsilon} d^3 k \frac{k^{\alpha\beta}}{(k^2)^2} = 0$ due to the symmetric integration in the loop momentum, and after using the identity

$$\int_{\epsilon} \frac{d^3 k}{(2\pi)^3} \frac{k^{\alpha\beta}}{[(k+p)^2 + m^2](k^2 + m^2)} = -\frac{p^{\alpha\beta}}{2} \int_{\epsilon} \frac{d^3 k}{(2\pi)^3} \frac{1}{[(k+p)^2 + m^2](k^2 + m^2)}, \tag{28}$$

equation (27) can be written as

$$\Gamma_4^{(2)} = \frac{N}{4} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta f(p)(-p^{\alpha\beta} + 2mC^{\alpha\beta})A_{\beta}(p, \theta)W_{\alpha}(-p, \theta), \tag{29}$$

where

$$W^{\alpha} = \frac{1}{2}D^{\beta}D^{\alpha}A_{\beta} = \frac{1}{2}D^{\alpha}D^{\beta}A_{\beta} + D^2A^{\alpha}, \tag{30}$$

corresponds to the linear part of the (noncommutative) Maxwell superfield strength. Using the relation $D^2W_{\alpha} = -p_{\alpha}^{\beta}W_{\beta}$ [33], we can write equation (29) as

$$\Gamma_{3a+3b}^{(2)} = \frac{N}{4} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta f(p)(W^{\alpha}W_{\alpha} + 2mA^{\alpha}W_{\alpha}), \tag{31}$$

where we omitted the explicit arguments of the external fields since they follow the same pattern as in the previous equations. In equation (31) the first and second terms are induced nonlocal Maxwell and Chern–Simons terms. The local limit obtained by the approximation $f(p) \simeq f(0) = -1/8\pi m$ gives for the coefficient of the induced Chern–Simons term the value $N/16\pi$.

To calculate the propagator of A^α we need to fix a gauge. One frequent choice in the literature is the Wess–Zumino gauge, which amounts to choosing the components χ_α and B of the superpotential A_α (see equation (A.1) in the appendix) as vanishing; this choice greatly simplifies the calculation in terms of component fields, but it breaks manifest supersymmetry and can lead into difficulties. So, we will fix the gauge by adding to the action given by equation (31) a covariant nonlocal gauge fixing term,

$$S_{GF} = \frac{N}{8\xi} \int \frac{d^3 p}{(2\pi)^3} d^2\theta f(p) D^\beta A_\beta D^2 D^\alpha A_\alpha, \tag{32}$$

and the corresponding Faddeev–Popov action,

$$S_{FP} = -\frac{N}{4} \int \frac{d^3 p}{(2\pi)^3} d^2\theta f(p) (c' D^2 c - ic' D^\alpha [A_\alpha, c]). \tag{33}$$

With this gauge choice, the part of the action quadratic in A^α turns out to be

$$\Gamma_A^{(2)} = -\frac{N}{8} \int \frac{d^3 p}{(2\pi)^3} d^2\theta f(p) A_\alpha \left[D^\beta D^\alpha (D^2 + 2m) + \frac{1}{\xi} D^\alpha D^\beta D^2 \right] A_\beta, \tag{34}$$

from which follows the propagator

$$\langle T A^\alpha(p, \theta_1) A^\beta(-p, \theta_2) \rangle = \frac{i}{Nf(p)} \left[\frac{(D^2 - 2m) D^\beta D^\alpha}{p^2(p^2 + 4m^2)} + \xi \frac{D^2 D^\alpha D^\beta}{p^4} \right] \delta_{12}, \tag{35}$$

or in another form, that will be useful for following calculations:

$$\begin{aligned} \langle T A^\alpha(p, \theta_1) A^\beta(-p, \theta_2) \rangle &= \frac{i}{Nf(p)} \left[-\frac{2mp^{\alpha\beta}}{p^2(p^2 + 4m^2)} + \left(\frac{1}{p^2 + 4m^2} - \frac{\xi}{p^2} \right) C^{\alpha\beta} \right. \\ &\quad \left. + \frac{p^{\alpha\beta}}{p^2} \left(\frac{1}{p^2 + 4m^2} + \frac{\xi}{p^2} \right) D^2 + \frac{2mC^{\alpha\beta}}{p^2(p^2 + 4m^2)} D^2 \right] \delta_{12}. \end{aligned} \tag{36}$$

The ghost propagator obtained from the Faddeev–Popov action is

$$\langle T c'(p, \theta_1) c(-p, \theta_2) \rangle = -i \frac{4\pi}{N} \frac{D^2}{f(p)p^2} \delta_{12}, \tag{37}$$

and its contribution only appears at the $1/N^2$ order, so that up to the order of approximation we consider, its contribution will not appear.

3.3. Is there an $A^\alpha \Sigma$ mixing?

From the action in equation (20), we see that a $1/N$ order process mixing A^α and Σ is in principle possible, what would result in a mixed propagator. The graph contributing to this process is represented in figure 5 and the corresponding Feynman amplitude,

$$\begin{aligned} \Gamma_{A\Sigma}^{(2)} &= \left\langle T : -i \int d^5 z \Sigma(z) \Phi_a(z) \bar{\Phi}_a(z) : \right. \\ &\quad \left. \times : \frac{i}{2!} \int d^5 z' \frac{i}{2} [D^\alpha \bar{\Phi}_b(z') A_\alpha(z') \Phi_b(z') + \bar{\Phi}_b(z') A_\alpha(z') D^\alpha \Phi_b(z')] : \right\rangle, \end{aligned} \tag{38}$$

can be separated into two terms, $\Gamma_{A\Sigma}^{(2)} = \Gamma_{A\Sigma,1}^{(2)} + \Gamma_{A\Sigma,2}^{(2)}$, the first one giving

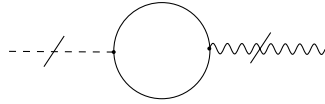


Figure 5. Supergraph that could imply into a leading-order process mixing the spinorial and the Σ superfields.

$$\Gamma_{A\Sigma,1}^{(2)} = i \frac{N}{4} \int \frac{d^3 p}{(2\pi)^3} d^2 \theta \int_{\epsilon} \frac{d^3 k}{(2\pi)^3} \frac{(k^{\beta\alpha} - m C^{\beta\alpha})}{[(k+p)^2 + m^2](k^2 + m^2)} D_{\beta} A_{\alpha}(-p, \theta) \Sigma(p, \theta), \quad (39)$$

while, for the second term, we found $\Gamma_{A\Sigma,2}^{(2)} = -\Gamma_{A\Sigma,1}^{(2)}$, so that the would be $\Gamma_{A\Sigma}^{(2)}$ vanishes. Essential to this result was the choice of the order of the fields $\Sigma\Phi_a\bar{\Phi}_a$, instead of $\Sigma\bar{\Phi}_a\Phi_a$, in the action in equation (20). This choice, nevertheless, requires Σ to change under gauge transformations, to keep the action gauge invariant, as we pointed out earlier. Clearly, one could choose the $\Sigma\bar{\Phi}\Phi$ order to keep Σ gauge invariant, but then the Moyal phase factors in the two vertices in equation (38) would not compensate each other and, as a consequence, $\Gamma_{A\Sigma,1}^{(2)}$ and $\Gamma_{A\Sigma,2}^{(2)}$ would not cancel. In this way, a mixed term $A\Sigma$ would be generated in the effective action, making highly cumbersome the evaluation of propagators and quantum corrections.

4. Renormalizability of the model

Let us now investigate the renormalizability of the model at the leading $1/N$ order. The power counting for the supersymmetric $CP^{(N-1)}$ model, in the fundamental representation, is the same as in the adjoint representation that was studied in [21], so we just quote the result. The superficial degree of divergence of a given supergraph is

$$\omega = 2 - \frac{1}{2} (E_{\Phi} + E_A + N_D) - E_{\Sigma} - E_c, \quad (40)$$

where E_i is the number of external legs of the field i , and N_D is the number of covariant derivatives acting on the external legs. We remark that, in order to have an iso-scalar contribution to the effective action, these variables are subjected to the constraints that E_{Φ} and the sum $E_A + N_D$ are even numbers.

From equation (40) we see that, apart from vacuum diagrams, the most divergent quantum corrections to the effective action are linearly ultraviolet divergent, and these can be dangerous to the renormalizability of the model since they can generate non-integrable (linear) infrared UV/IR singularities. It is essential to secure that linear UV/IR singularities do not appear since they would invalidate the $1/N$ expansion at higher orders [10]. Some of the graphs with $\omega = 1$ have already been analysed and shown not to generate dangerous linear divergences: the ones corresponding to the spinorial superfield effective action ($E_A = 2$), calculated in section 3.2, and the graph with $E_{\Sigma} = 1$, which has been taken into account in the gap equations. The only remaining contributions with $\omega = 1$ is the one with $(E_A, N_D) = (1, 1)$, which vanishes; in fact, it is proportional to $\int d^2\theta_1 D^{\alpha}(D^2 + 2m)\delta_{11} = 0$. Since there is no corresponding counterterm in S_{CT} , the finiteness of this contribution is essential to the renormalizability of the model.

Now, we focus on graphs with logarithmic power counting. These generate integrable, and therefore harmless, UV/IR infrared singularities. However, some of these graphs are still dangerous because they can generate ultraviolet divergent contributions to the effective action which do not have corresponding counterterms in S_{CT} . Listing all possible graphs with $\omega = 0$, one finds several such potentially dangerous corrections. The contribution with

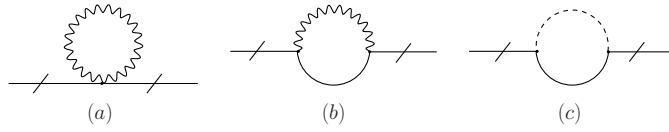


Figure 6. Supergraphs contributing to the $1/N$ subleading contributions to the quadratic effective action of the Φ superfield.

$E_\Sigma = 2$ has been already analysed and found to be finite in section 3.1, whereas the one with $(E_A, N_D, E_\Sigma) = (1, 1, 1)$ (the $A\Sigma$ mixing) yields a vanishing result, as shown in section 3.3. It still remains some harmful possibilities, namely for $E_A = 4, E_\Phi = 4, (E_A, E_\Sigma) = (2, 1)$ and $(E_A, N_D) = (3, 1)$. However, after checking that the phase factor induced by the Moyal product is planar in all these cases, one can argue that the logarithmically divergent parts of the Feynman integrals will be proportional to $\int_\epsilon d^3k \frac{k^{\alpha\beta}}{(k^2)^2}$, which vanishes due to the symmetric integration in the loop momentum.

As for the remaining logarithmically divergent supergraphs, they correspond to terms present in the counterterm action and are, therefore, in principle renormalizable. An explicit verification of the renormalizability of the supersymmetric noncommutative $CP^{(N-1)}$ model would involve the calculation of the subleading corrections to several vertex functions, and one example of such a calculation is presented in the next section.

5. Subleading corrections to the $\bar{\Phi}\Phi$ effective action

Let us calculate in detail the subleading contributions to the quadratic effective action of the Φ superfield, which arise from the diagrams depicted in figure 6. Here the calculations become quite involved, and part of them were performed with the help of a symbolic computer program designed for superfield calculations [45]. The Feynman amplitude corresponding to the graph in figure 6(a) is

$$\Gamma_{6a}^{(2)} = -\frac{i}{2} \left(\sum_{j=1}^4 \int \frac{d^3k_j}{(2\pi)^3} \right) \int d^2\theta (2\pi)^3 \delta^3(k_1 + k_2 + k_3 + k_4) (2\pi)^3 \delta^3(k_2 + k_3) C^{\alpha\beta} \times \exp\{-i[k_2 \wedge (k_3 + k_4) + k_3 \wedge k_4]\} \langle T A_\beta(k_2, \theta) A_\alpha(k_3, \theta) \bar{\Phi}_a(k_1) \Phi_a(k_4) \rangle. \quad (41)$$

Integrating in k_3 and k_4 , renaming $k_1 = p, k_2 = k$ and using that $\delta(\theta - \theta) = 0$, we arrive at

$$\Gamma_{6a}^{(2)} = \frac{1}{N} \int \frac{d^3p}{(2\pi)^3} d^2\theta \bar{\Phi}_a(-p, \theta) \Phi_a(p, \theta) \int \frac{d^3k}{(2\pi)^3} \frac{-2m}{f(k)k^2(k^2 + 4m^2)}. \quad (42)$$

The amplitude corresponding to figure 6(b) is given by

$$\Gamma_{6b}^{(2)} = \frac{1}{N} \int \frac{d^3p}{(2\pi)^3} d^2\theta \bar{\Phi}_a(-p, \theta) \int \frac{d^3k}{(2\pi)^3} \frac{1}{f(k)[(k+p)^2 + m^2]} \times \left\{ \frac{k^{\alpha\beta} p_{\alpha\beta}}{k^2(k^2 + 4m^2)} (D^2 + m) + \xi \frac{k^{\alpha\beta} p_{\alpha\beta}}{k^4} (D^2 - m) + \frac{2D^2}{(k^2 + 4m^2)} + \xi \frac{1}{k^2} (D^2 - m) - \frac{4m}{k^2(k^2 + 4m^2)} D^2 (D^2 - m) \right\} \Phi_a(p, \theta), \quad (43)$$

and finally the contribution of figure 6(c) is

$$\Gamma_{6c}^{(2)} = -\frac{1}{N} \int \frac{d^3p}{(2\pi)^3} d^2\theta \int \frac{d^3k}{(2\pi)^3} \frac{(D^2 - m) \Phi_a(p, \theta) \bar{\Phi}_a(-p, \theta)}{f(k)[(k+p)^2 + m^2](k^2 + m^2)}. \quad (44)$$

By adding the above contributions, we get for the radiative corrections to the quadratic action in Φ the expression

$$\Gamma_6^{(2)} = \frac{1}{N} \int \frac{d^3 p}{(2\pi)^3} d^2\theta \bar{\Phi}(-p, \theta) \int_\epsilon \frac{d^3 k}{(2\pi)^3} \frac{1}{f(k)} \left\{ \frac{(D^2 - m)^2}{k^2(k^2 + 4m^2)[(k+p)^2 + m^2]} + \frac{1}{k^2(k^2 + 4m^2)} + \xi \frac{k^2 + 2k \cdot p}{k^4[(k+p)^2 + m^2]} \right\} (D^2 - m)\Phi(p, \theta), \quad (45)$$

which is planar and, therefore, do not generate UV/IR mixing. Despite being non-local, it still shares an overall $(D^2 - m)$ factor with the piece of the classical action quadratic in Φ . Separating its logarithmic divergent part,

$$\Gamma_6^{(2)} = \frac{1}{N} \left[(1 + \xi) \int_\epsilon \frac{d^3 k}{(2\pi)^4} \frac{1}{f(k)} \frac{1}{k^2(k^2 + m^2)} + \text{finite terms} \right] \times \int \frac{d^3 p}{(2\pi)^3} d^2\theta \bar{\Phi}_a(-p, \theta) (D^2 - m)\Phi_a(p, \theta) \quad (46)$$

we see that a wavefunction renormalization $\Phi \rightarrow \sqrt{Z}\Phi$ takes care of the logarithmic UV divergence in equation (46); additionally, $\Gamma_6^{(2)}$ is finite in the gauge $\xi = -1$. The important point we want to stress is that we were able to renormalize this vertex function in an explicitly supersymmetric fashion, which is not the case if one works in the Wess–Zumino gauge [5], when the bosonic and fermionic components of the superfield Φ receives different wavefunction renormalizations.

As for the other logarithmically divergent contributions, their calculation is much more complicated, involving graphs with one and two loops of momentum. A complete evaluation of such subleading contributions is out of the scope of this work, but in such a calculation the graphical identity represented in figure 3 would be essential to secure the cancellation of several UV divergences. Indeed, in the usual (commutative, nonsupersymmetric) $CP^{(N-1)}$ model, the proof of renormalizability at arbitrary order of the $1/N$ expansion heavily relies on such identity [3].

6. Conclusions

In this paper, we studied the noncommutative supersymmetric $CP^{(N-1)}$ model in $2 + 1$ spacetime dimensions, when the basic field is in the fundamental representation. We found that the model has the same phase structure as its commutative counterpart. Differently from the previous analysis in [21], in this case the model classically goes, in the $\Theta \rightarrow 0$ limit, to the usual (commutative) supersymmetric $CP^{(N-1)}$ model. At the quantum level, the UV/IR mixing generates only mild infrared singularities, so that renormalizability at the leading $1/N$ order is explicitly checked.

The use of the superspace approach ensures a manifestly supersymmetric renormalization, which is not necessarily the case in the components fields formalism. In [5], where the ultraviolet behaviour of the commutative supersymmetric $CP^{(N-1)}$ model was considered, the scalar and fermionic superpartners received different renormalizations, so that the supersymmetric invariance of the quantum theory becomes non-manifest. This problem does not appear in the superfield formalism.

We have also studied the first subleading correction to the effective action of the Φ field, and shown that it can be made finite by only a wavefunction renormalization, as it should. Such explicit calculations have not appeared in the literature so far, even in the commutative case.

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Appendix

For the sake of clarity, we will show how the action of the noncommutative supersymmetric $CP^{(N-1)}$ model looks like when written in terms of component fields. We define the components of the spinor gauge superpotential as

$$\begin{aligned}\chi_\alpha(x) &= A_\alpha(\theta, x)|, \\ B(x) &= \frac{1}{2}D^\alpha A_\alpha(\theta, x)|, \\ V_{\alpha\beta}(x) &= -\frac{i}{2}(D_\alpha A_\beta + D_\beta A_\alpha)(\theta, x)|, \\ \lambda_\alpha(x) &= \frac{1}{2}D^\beta D_\alpha A_\beta(\theta, x)|,\end{aligned}\tag{A.1}$$

and the components of the fields Φ and Σ in gauge covariant way through,

$$\phi(x) = \Phi(x, \theta)|, \quad \psi_\alpha(x) = \nabla_\alpha \Phi(x, \theta)|, \quad F(x) = \nabla^2 \Phi(x, \theta)|,\tag{A.2}$$

and

$$\kappa(x) = \Sigma(x, \theta)|, \quad \zeta_\alpha(x) = \nabla_\alpha \Sigma(x, \theta)|, \quad \sigma(x) = \nabla^2 \Sigma(x, \theta)|,\tag{A.3}$$

where the instruction $|$ means to take $\theta = 0$ after doing the derivatives. Using these definitions, the action in equation (3) can be cast as

$$\begin{aligned}S = \int d^3x \left\{ \bar{F}_a F_a + \bar{\psi}_a^\alpha (i\partial_\alpha^\beta + V_\alpha^\beta) \psi_{a\beta} + (i\bar{\psi}_a^\alpha \lambda_\alpha \phi_a + \text{c.h.}) \right. \\ \left. + (\partial_{\alpha\beta} \bar{\phi}_a + i\bar{\phi}_a V_{\alpha\beta})(\partial_{\alpha\beta} \phi_a - iV_{\alpha\beta} \phi_a) - \sigma \left(\phi_a \bar{\phi}_a - \frac{N}{g_0} \right) \right. \\ \left. - \zeta^\alpha (\psi_{a\alpha} \bar{\phi}_a + \phi_a \bar{\psi}_{a\alpha}) - \kappa (F_a \bar{\phi}_a + \phi_a \bar{F}_a + \psi_a^\alpha \bar{\psi}_{a\alpha}) \right\}.\end{aligned}\tag{A.4}$$

As in the main text of this paper, the Moyal product is not written explicitly in this and in the remaining formulae. The auxiliary fields F and \bar{F} can be eliminated by means of their equations of motion and, in this way, equation (A.4) is reduced to

$$\begin{aligned}S = \int d^3x \left\{ \bar{\psi}_a^\alpha (i\partial_\alpha^\beta + V_\alpha^\beta) \psi_{a\beta} + (\partial_{\alpha\beta} \bar{\phi}_a + i\bar{\phi}_a V_{\alpha\beta})(\partial_{\alpha\beta} \phi_a - iV_{\alpha\beta} \phi_a) \right. \\ \left. - \sigma \left(\phi_a \bar{\phi}_a - \frac{N}{g_0} \right) - \bar{\phi}_a (\zeta^\alpha + i\lambda^\alpha) \psi_{a\alpha} - \bar{\psi}_{a\alpha} (\zeta^\alpha - i\lambda^\alpha) \phi_a \right. \\ \left. - \kappa^2 \phi_a \bar{\phi}_a - \kappa \psi_a^\alpha \bar{\psi}_{a\alpha} \right\}.\end{aligned}\tag{A.5}$$

Finally, by writing the bi-spinors in terms of the more usual 3-vectors through $V_{\alpha\beta} = 1/2(\gamma^\mu)_{\alpha\beta} A_\mu$ and $\partial_{\alpha\beta} = 1/2(\gamma^\mu)_{\alpha\beta} \partial_\mu$, where γ^μ are the three 2×2 Dirac matrices in 2 + 1 dimensions, we arrive at

$$S = \int d^3x \left\{ \bar{\phi}_a \square \phi_a - \sigma \left(\phi_a \bar{\phi}_a - \frac{N}{g_0} \right) - i \left(\bar{\phi}_a \overleftrightarrow{\partial}^\mu \phi_a \right) A_\mu + \bar{\phi}_a A^2 \phi_a \right\}$$

$$\begin{aligned}
& + i\bar{\psi}_a \gamma^\mu (\partial_\mu - iA_\mu) \psi_a - \bar{\phi}_a (\zeta + i\lambda) \psi_a - \bar{\psi}_a (\zeta - i\lambda) \phi_a \\
& - \kappa^2 \phi_a \bar{\phi}_a - \kappa \psi_a \bar{\psi}_a \Big\}. \tag{A.6}
\end{aligned}$$

This expression shows more explicitly what is the theory we are working with, when dealing with the more compact superfield notation. One can realize that the first line corresponds to the bosonic model of equation (1), extended to the noncommutative spacetime, but we now have the additional fermionic degrees of freedom, necessary for supersymmetry, and a new constraint imposed by the combination $\zeta + i\lambda$, and κ is a composite field classically given by $\kappa = -\psi_a \bar{\psi}_a / 2\phi_a \bar{\phi}_a$. Elimination of κ in equation (A.6) yields a four-fermion self-interaction, typical of the supersymmetric extension of the $CP^{(N-1)}$ model [4].

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